

# UNDERSTANDING FLAVOR MIXING IN QUANTUM FIELD THEORY

M. BLASONE

*Blackett Laboratory, Imperial College,  
Prince Consort Road, London SW7 2BZ, U.K.  
E-mail: m.blasone@ic.ac.uk*

A. CAPOLUPO, G. VITIELLO

*Dipartimento di Fisica and INFN,  
Università di Salerno, I-84100 Salerno, Italy  
E-mails: capolupo@sa.infn.it, vitiello@sa.infn.it*

## Abstract

We report on recent results showing that a rich non-perturbative vacuum structure is associated with flavor mixing in Quantum Field Theory. We treat explicitly the case of mixing among three generations of Dirac fermions. Exact oscillation formulas are presented exhibiting new features with respect to the usual ones. CP and T violation are also discussed.

## 1 Introduction

In recent years, there has been much progress in the understanding of flavor mixing in Quantum Field Theory (QFT). The original discovery of the unitary inequivalence of the mass and the flavor representations in QFT<sup>1</sup>, has prompted further investigations on fermion mixing<sup>2,3,4,5,6,7</sup> as well as on boson mixing<sup>8,9,10,11</sup>. It has emerged that the rich non-perturbative vacuum structure associated with field mixing, leads to phenomenologically relevant modification of the flavor oscillation formulas, exhibiting new features with respect to the usual quantum-mechanical ones<sup>12</sup>.

In the following we discuss three flavor fermion mixing explicitly. A discussion of mixing of boson fields can be found in Ref.<sup>10</sup>.

## 2 Three flavor fermion mixing

Let us consider the following Lagrangian density, describing three free Dirac fields with a mixed mass term (in the following we will refer explicitly to

neutrinos):

$$\mathcal{L}(x) = \bar{\Psi}_f(x) (i \not{\partial} - M) \Psi_f(x), \quad (1)$$

where  $\Psi_f^T = (\nu_e, \nu_\mu, \nu_\tau)$  and  $M^\dagger = M$ .

Among the various possible parameterizations of the three fields mixing matrix, we choose to work with the CKM matrix<sup>13</sup>:

$$\Psi_f(x) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \Psi_m(x) \quad (2)$$

with  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$  being  $\theta_{ij}$  the mixing angles,  $\delta$  the CP-violating phase and  $\Psi_m^T = (\nu_1, \nu_2, \nu_3)$ .

Using Eq.(2), we diagonalize Eq.(1) to the Lagrangian for three Dirac fields, with definite masses:

$$\mathcal{L}(x) = \bar{\Psi}_m(x) (i \not{\partial} - M_d) \Psi_m(x), \quad (3)$$

where  $M_d = \text{diag}(m_1, m_2, m_3)$ . To generate the mixing matrix Eq.(2), we define<sup>1</sup>:

$$G_{12}(t) = e^{\theta_{12}L_{12}(t)} \quad , \quad G_{23}(t) = e^{\theta_{23}L_{23}(t)} \quad , \quad G_{13}(t) = e^{\theta_{13}L_{13}(\delta, t)}, \quad (4)$$

where

$$\begin{aligned} L_{12}(t) &= \int d^3x (\nu_1^\dagger(x) \nu_2(x) - \nu_2^\dagger(x) \nu_1(x)), \\ L_{23}(t) &= \int d^3x (\nu_2^\dagger(x) \nu_3(x) - \nu_3^\dagger(x) \nu_2(x)), \\ L_{13}(\delta, t) &= \int d^3x (\nu_1^\dagger(x) \nu_3(x) e^{-i\delta} - \nu_3^\dagger(x) \nu_1(x) e^{i\delta}). \end{aligned} \quad (5)$$

We can thus write Eq.(2) in the form:

$$\nu_\sigma^\alpha(x) = G_\theta^{-1}(t) \nu_i^\alpha(x) G_\theta(t), \quad (6)$$

with  $(\sigma, i) = (e, 1), (\mu, 2), (\tau, 3)$  and

$$G_\theta(t) \equiv G_{23}(t)G_{13}(t)G_{12}(t). \quad (7)$$

From Eqs.(4)-(7), we see that the phase  $\delta$  is unavoidable for three fields mixing, while it can be incorporated in the definition of the fields in the case of two

flavors. Note also that other parameterizations of the mixing matrix can be reproduced by means of the generators Eq.(4), which do not commute among themselves (see below).

The free fields  $\nu_i$  can be quantized in the usual way<sup>14</sup> (we use  $t \equiv x_0$ ):

$$\nu_i(x) = \sum_r \int d^3k \left[ u_{\mathbf{k},i}^r(t) \alpha_{\mathbf{k},i}^r + v_{-\mathbf{k},i}^r(t) \beta_{-\mathbf{k},i}^{r\dagger} \right] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad i = 1, 2, 3 \quad (8)$$

with  $\omega_{k,i} = \sqrt{\mathbf{k}^2 + m_i^2}$ . The vacuum for the mass eigenstates is denoted by  $|0\rangle_m$ :  $\alpha_{\mathbf{k},i}^r|0\rangle_m = \beta_{\mathbf{k},i}^r|0\rangle_m = 0$ . The anticommutation relations are the usual ones; the wave function orthonormality and completeness relations are those of Ref.<sup>1</sup>.

By use of  $G_\theta(t)$ , the flavor fields can be expanded as:

$$\nu_\sigma(x) = \sum_r \int d^3k \left[ u_{\mathbf{k},i}^r(t) \alpha_{\mathbf{k},\sigma}^r(t) + v_{-\mathbf{k},i}^r(t) \beta_{-\mathbf{k},\sigma}^{r\dagger}(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}.$$

with  $(\sigma, i) = (e, 1), (\mu, 2), (\tau, 3)$  and  $\alpha_{\mathbf{k},\sigma}^r(t) \equiv G_\theta^{-1}(t) \alpha_{\mathbf{k},i}^r G_\theta(t)$  and  $\beta_{-\mathbf{k},\sigma}^{r\dagger}(t) \equiv G_\theta^{-1}(t) \beta_{-\mathbf{k},i}^{r\dagger} G_\theta(t)$ .

The crucial point<sup>1</sup>, is that the generator of the field mixing Eq.(2) does not annihilate the vacuum for the free fields. We are thus lead to define a new state, the *flavor vacuum*, in the following way:

$$|0(t)\rangle_f \equiv G_\theta^{-1}(t) |0\rangle_m. \quad (9)$$

The unitary inequivalence (in the infinite volume limit) of  $|0\rangle_m$  with  $|0(t)\rangle_f$  has been rigorously proved for an arbitrary number of generations<sup>5</sup>.

The explicit form of the above defined flavor annihilation operators (in the reference frame  $\mathbf{k} = (0, 0, |\mathbf{k}|)$ ) is

$$\begin{aligned} \alpha_{\mathbf{k},e}^r(t) = & c_{12}c_{13} \alpha_{\mathbf{k},1}^r + s_{12}c_{13} \left( U_{12}^{\mathbf{k}*} \alpha_{\mathbf{k},2}^r + \epsilon^r V_{12}^{\mathbf{k}} \beta_{-\mathbf{k},2}^{r\dagger} \right) \\ & + e^{-i\delta} s_{13} \left( U_{13}^{\mathbf{k}*} \alpha_{\mathbf{k},3}^r + \epsilon^r V_{13}^{\mathbf{k}} \beta_{-\mathbf{k},3}^{r\dagger} \right), \end{aligned} \quad (10)$$

$$\begin{aligned} \alpha_{\mathbf{k},\mu}^r(t) = & (c_{12}c_{23} - e^{i\delta} s_{12}s_{23}s_{13}) \alpha_{\mathbf{k},2}^r + s_{23}c_{13} \left( U_{23}^{\mathbf{k}*} \alpha_{\mathbf{k},3}^r + \epsilon^r V_{23}^{\mathbf{k}} \beta_{-\mathbf{k},3}^{r\dagger} \right) \\ & - (s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}) \left( U_{12}^{\mathbf{k}} \alpha_{\mathbf{k},1}^r - \epsilon^r V_{12}^{\mathbf{k}} \beta_{-\mathbf{k},1}^{r\dagger} \right), \end{aligned} \quad (11)$$

$$\begin{aligned} \alpha_{\mathbf{k},\tau}^r(t) = & c_{23}c_{13} \alpha_{\mathbf{k},3}^r - (c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}) \left( U_{23}^{\mathbf{k}} \alpha_{\mathbf{k},2}^r - \epsilon^r V_{23}^{\mathbf{k}} \beta_{-\mathbf{k},2}^{r\dagger} \right) \\ & + (s_{12}s_{23} - e^{i\delta} c_{12}c_{23}s_{13}) \left( U_{13}^{\mathbf{k}} \alpha_{\mathbf{k},1}^r - \epsilon^r V_{13}^{\mathbf{k}} \beta_{-\mathbf{k},1}^{r\dagger} \right), \end{aligned} \quad (12)$$

$$\begin{aligned}\beta_{-\mathbf{k},e}^r(t) &= c_{12}c_{13} \beta_{-\mathbf{k},1}^r + s_{12}c_{13} \left( U_{12}^{\mathbf{k}*} \beta_{-\mathbf{k},2}^r - \epsilon^r V_{12}^{\mathbf{k}} \alpha_{\mathbf{k},2}^{r\dagger} \right) \\ &\quad + e^{i\delta} s_{13} \left( U_{13}^{\mathbf{k}*} \beta_{-\mathbf{k},3}^r - \epsilon^r V_{13}^{\mathbf{k}} \alpha_{\mathbf{k},3}^{r\dagger} \right),\end{aligned}\quad (13)$$

$$\begin{aligned}\beta_{-\mathbf{k},\mu}^r(t) &= (c_{12}c_{23} - e^{-i\delta} s_{12}s_{23}s_{13}) \beta_{-\mathbf{k},2}^r + s_{23}c_{13} \left( U_{23}^{\mathbf{k}*} \beta_{-\mathbf{k},3}^r - \epsilon^r V_{23}^{\mathbf{k}} \alpha_{\mathbf{k},3}^{r\dagger} \right) \\ &\quad - (s_{12}c_{23} + e^{-i\delta} c_{12}s_{23}s_{13}) \left( U_{12}^{\mathbf{k}} \beta_{-\mathbf{k},1}^r + \epsilon^r V_{12}^{\mathbf{k}} \alpha_{\mathbf{k},1}^{r\dagger} \right),\end{aligned}\quad (14)$$

$$\begin{aligned}\beta_{-\mathbf{k},\tau}^r(t) &= c_{23}c_{13} \beta_{-\mathbf{k},3}^r - (c_{12}s_{23} + e^{-i\delta} s_{12}c_{23}s_{13}) \left( U_{23}^{\mathbf{k}} \beta_{-\mathbf{k},2}^r + \epsilon^r V_{23}^{\mathbf{k}} \alpha_{\mathbf{k},2}^{r\dagger} \right) \\ &\quad + (s_{12}s_{23} - e^{-i\delta} c_{12}c_{23}s_{13}) \left( U_{13}^{\mathbf{k}} \beta_{-\mathbf{k},1}^r + \epsilon^r V_{13}^{\mathbf{k}} \alpha_{\mathbf{k},1}^{r\dagger} \right).\end{aligned}\quad (15)$$

with

$$V_{ij}^{\mathbf{k}} = |V_{ij}^{\mathbf{k}}| e^{i(\omega_{k,j} + \omega_{k,i})t}, \quad U_{ij}^{\mathbf{k}} = |U_{ij}^{\mathbf{k}}| e^{i(\omega_{k,j} - \omega_{k,i})t} \quad (16)$$

$$|U_{ij}^{\mathbf{k}}| = \cos \xi_{ij}^{\mathbf{k}}, \quad |V_{ij}^{\mathbf{k}}| = \sin \xi_{ij}^{\mathbf{k}} \quad (17)$$

$$\xi_{ij}^{\mathbf{k}} = \arctan \left[ \frac{k(\omega_{k,i} - \omega_{k,j} + m_i - m_j)}{|k|^2 + (\omega_{k,i} + m_i)(\omega_{k,j} + m_j)} \right] \quad (18)$$

where  $i, j = 1, 2, 3$  and  $j > i$ . The above operators annihilate  $|0(t)\rangle_f$  and satisfy canonical anticommutation relations (at equal times). Note that

$$|U_{ij}^{\mathbf{k}}| \rightarrow 1, \quad |V_{ij}^{\mathbf{k}}| \rightarrow 0 \quad \text{for } |k| \gg \sqrt{m_i m_j}, \quad (19)$$

### 3 Currents and charges for three flavor fermion mixing

Let us now consider the transformations acting on the triplet of free fields with different masses. The Lagrangian Eq.(3) is invariant under global  $U(1)$  phase transformations of the type  $\Psi'_m = e^{i\alpha} \Psi_m$ : as a result, we have the conservation of the Noether charge  $Q = \int d^3x I^0(x)$ , with  $I^\mu(x) = \bar{\Psi}_m(x) \gamma^\mu \Psi_m(x)$ , which is indeed the total charge of the system (i.e. the total lepton number).

Consider now the  $SU(3)$  transformations acting on  $\Psi_m$ :

$$\Psi'_m(x) = e^{i\alpha_j F_j} \Psi_m(x), \quad j = 1, 2, \dots, 8. \quad (20)$$

with  $\alpha_j$  real constants,  $F_j = \lambda_j/2$  and  $\lambda_j$  being the Gell-Mann matrices. For  $m_1 \neq m_2 \neq m_3$ , the Lagrangian is not generally invariant under (20) and we

obtain, by use of the equations of motion,

$$\begin{aligned}\delta\mathcal{L}(x) &= i\alpha_j \bar{\Psi}_m(x) [F_j, M_d] \Psi_m(x) = -\alpha_j \partial_\mu J_{m,j}^\mu(x) \\ J_{m,j}^\mu(x) &= \bar{\Psi}_m(x) \gamma^\mu F_j \Psi_m(x) \quad , \quad j = 1, 2, \dots, 8.\end{aligned}\quad (21)$$

The explicit form of the currents is:

$$\begin{aligned}J_{m,1}^\mu &= \frac{1}{2} [\bar{\nu}_1 \gamma^\mu \nu_2 + \bar{\nu}_2 \gamma^\mu \nu_1] \quad , \quad J_{m,2}^\mu = -\frac{i}{2} [\bar{\nu}_1 \gamma^\mu \nu_2 - \bar{\nu}_2 \gamma^\mu \nu_1] \\ J_{m,3}^\mu &= \frac{1}{2} [\bar{\nu}_1 \gamma^\mu \nu_1 - \bar{\nu}_2 \gamma^\mu \nu_2] \quad , \quad J_{m,4}^\mu = \frac{1}{2} [\bar{\nu}_1 \gamma^\mu \nu_3 + \bar{\nu}_3 \gamma^\mu \nu_1] \\ J_{m,5}^\mu &= -\frac{i}{2} [\bar{\nu}_1 \gamma^\mu \nu_3 - \bar{\nu}_3 \gamma^\mu \nu_1] \quad , \quad J_{m,6}^\mu = \frac{1}{2} [\bar{\nu}_2 \gamma^\mu \nu_3 + \bar{\nu}_3 \gamma^\mu \nu_2] \\ J_{m,7}^\mu &= -\frac{i}{2} [\bar{\nu}_2 \gamma^\mu \nu_3 - \bar{\nu}_3 \gamma^\mu \nu_2] \quad , \quad J_{m,8}^\mu = \frac{1}{2\sqrt{3}} [\bar{\nu}_1 \gamma^\mu \nu_1 + \bar{\nu}_2 \gamma^\mu \nu_2 - 2\bar{\nu}_3 \gamma^\mu \nu_3].\end{aligned}\quad (22)$$

The charges  $Q_{m,j}(t) \equiv \int d^3x J_{m,j}^0(x)$ , satisfy the  $SU(3)$  algebra at equal times:  $[Q_{m,j}(t), Q_{m,k}(t)] = i f_{jkl} Q_{m,l}(t)$ , with  $f_{jkl}$  totally antisymmetric. From (21) we see that  $Q_{m,3}$  and  $Q_{m,8}$  are conserved as  $M_d$  is diagonal. We can define the combinations:

$$\begin{aligned}Q_1 &\equiv \frac{1}{3}Q + Q_{m,3} + \frac{1}{\sqrt{3}}Q_{m,8}, \\ Q_2 &\equiv \frac{1}{3}Q - Q_{m,3} + \frac{1}{\sqrt{3}}Q_{m,8}, \\ Q_3 &\equiv \frac{1}{3}Q - \frac{2}{\sqrt{3}}Q_{m,8},\end{aligned}\quad (23)$$

$$Q_i = \sum_r \int d^3k \left( \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r - \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right) \quad , \quad i = 1, 2, 3. \quad (24)$$

These are nothing but the Noether charges associated with the non-interacting fields  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ : in the absence of mixing, they are the flavor charges, separately conserved for each generation. The generator of the mixing transformations can be now written as:

$$G_\theta(t) = e^{i2\theta_{23} Q_{m,7}(t)} e^{i2\theta_{13} Q_{m,5}(t)} e^{i2\theta_{12} Q_{m,2}(t)} \quad (25)$$

Performing  $SU(3)$  transformations on the flavor triplet  $\Psi_f$  gives a similar structure for the currents as before. The related charges  $Q_{f,j}(t) \equiv \int d^3x J_{f,j}^0(x)$  still close the  $SU(3)$  algebra. Due to the off-diagonal (mixing) terms in the

mass matrix  $M$ ,  $Q_{f,3}(t)$  and  $Q_{f,8}(t)$  are time-dependent. This implies an exchange of charge between  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ , resulting in the flavor oscillations.

In accordance with Eq.(23), we now define the *flavor charges* for mixed fields as

$$\begin{aligned} Q_e(t) &\equiv \frac{1}{3}Q + Q_{f,3}(t) + \frac{1}{\sqrt{3}}Q_{f,8}(t), \\ Q_\mu(t) &\equiv \frac{1}{3}Q - Q_{f,3}(t) + \frac{1}{\sqrt{3}}Q_{f,8}(t), \\ Q_\tau(t) &\equiv \frac{1}{3}Q - \frac{2}{\sqrt{3}}Q_{f,8}(t). \end{aligned} \quad (26)$$

with  $Q_e(t) + Q_\mu(t) + Q_\tau(t) = Q$ . These charges have a simple expression in terms of the flavor ladder operators ( $\sigma = e, \mu, \tau$ ):

$$Q_\sigma(t) = \sum_r \int d^3k \left( \alpha_{\mathbf{k},\sigma}^{r\dagger}(t) \alpha_{\mathbf{k},\sigma}^r(t) - \beta_{-\mathbf{k},\sigma}^{r\dagger}(t) \beta_{-\mathbf{k},\sigma}^r(t) \right), \quad (27)$$

since they are connected to the Noether charges  $Q_i$  of Eq.(23) via the mixing generator:  $Q_\sigma(t) = G_\theta^{-1}(t) Q_i G_\theta(t)$ .

#### 4 Neutrino oscillations

The oscillation formulas are obtained by taking expectation values of the above charges on the (flavor) neutrino state. Consider for example a  $\rho$ -flavor neutrino state defined as  $|\nu_\rho\rangle \equiv \alpha_{\mathbf{k},\rho}^{r\dagger}(0)|0\rangle_f$  (for a discussion on the correct definition of flavor states see Refs.<sup>2,3,10</sup>). Working in the Heisenberg picture, we obtain:

$$\begin{aligned} \mathcal{Q}_{\mathbf{k},\sigma}^\rho(t) &\equiv \langle \nu_\rho | Q_\sigma(t) | \nu_\rho \rangle - {}_f\langle 0 | Q_\sigma(t) | 0 \rangle_f \\ &= \left| \left\{ \alpha_{\mathbf{k},\sigma}^r(t), \alpha_{\mathbf{k},\rho}^{r\dagger}(0) \right\} \right|^2 + \left| \left\{ \beta_{-\mathbf{k},\sigma}^{r\dagger}(t), \alpha_{\mathbf{k},\rho}^{r\dagger}(0) \right\} \right|^2, \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{Q}_{\mathbf{k},\sigma}^{\bar{\rho}}(t) &\equiv \langle \bar{\nu}_\rho | Q_\sigma(t) | \bar{\nu}_\rho \rangle - {}_f\langle 0 | Q_\sigma(t) | 0 \rangle_f \\ &= - \left| \left\{ \beta_{\mathbf{k},\sigma}^r(t), \beta_{\mathbf{k},\rho}^{r\dagger}(0) \right\} \right|^2 - \left| \left\{ \alpha_{-\mathbf{k},\sigma}^{r\dagger}(t), \beta_{\mathbf{k},\rho}^{r\dagger}(0) \right\} \right|^2, \end{aligned} \quad (29)$$

where  $|0\rangle_f \equiv |0(0)\rangle_f$  and  $|\bar{\nu}_\rho\rangle \equiv \beta_{\mathbf{k},\rho}^{r\dagger}(0)|0\rangle_f$ . Charge conservation is obviously ensured at any time:  $\sum_\sigma \mathcal{Q}_{\mathbf{k},\sigma}(t) = 1$ . We remark that the expectation value of  $Q_\sigma$  cannot be taken on vectors of the Fock space built on  $|0\rangle_m$ , as shown

in Refs.<sup>2,3,10</sup>. Note that, in comparison with the two-flavor mixing, a vacuum contribution needs to be subtracted here: due to the presence of the CP violating phase  $\delta$ , we have indeed  ${}_f\langle 0|Q_\sigma(t)|0\rangle_f \neq 0$ .

The oscillation formulas for the flavor charges, on an initial electron neutrino state, then follow:

$$\begin{aligned} \mathcal{Q}_{\mathbf{k},e}^e(t) = & 1 - \sin^2 2\theta_{12} \cos^4 \theta_{13} \left[ \cos^2 \xi_{12}^{\mathbf{k}} \sin^2(\Delta_{12}^{\mathbf{k}} t) + \sin^2 \xi_{12}^{\mathbf{k}} \sin^2(\Omega_{12}^{\mathbf{k}} t) \right] \\ & - \sin^2 2\theta_{13} \cos^2 \theta_{12} \left[ \cos^2 \xi_{13}^{\mathbf{k}} \sin^2(\Delta_{13}^{\mathbf{k}} t) + \sin^2 \xi_{13}^{\mathbf{k}} \sin^2(\Omega_{13}^{\mathbf{k}} t) \right] \\ & - \sin^2 2\theta_{13} \sin^2 \theta_{12} \left[ \cos^2(\xi_{12}^{\mathbf{k}} - \xi_{13}^{\mathbf{k}}) \sin^2(\Delta_{23}^{\mathbf{k}} t) + \sin^2(\xi_{12}^{\mathbf{k}} - \xi_{13}^{\mathbf{k}}) \sin^2(\Omega_{23}^{\mathbf{k}} t) \right], \end{aligned} \quad (30)$$

and similar ones, with  $\Delta_{ij}^{\mathbf{k}} \equiv (\omega_{k,j} - \omega_{k,i})/2$  and  $\Omega_{ij}^{\mathbf{k}} \equiv (\omega_{k,j} + \omega_{k,i})/2$ . These results are exact. The differences with respect to the usual formulas for neutrino oscillations are in the energy dependence of the amplitudes and in the additional oscillating terms. In the relativistic limit of Eq.(19), the traditional QM (Pontecorvo) formulas are recovered.

## 5 CP and T violation in QFT neutrino oscillations

The CP violation developed in neutrino oscillations is given in QM as:

$$\Delta_{CP}(t) = P_{\nu_\sigma \rightarrow \nu_\rho}(t) - P_{\bar{\nu}_\sigma \rightarrow \bar{\nu}_\rho}(t). \quad (31)$$

where  $\sigma, \rho = e, \mu, \tau$ . The T violation can be obtained as:

$$\Delta_T(t) = P_{\nu_\sigma \rightarrow \nu_\rho}(t) - P_{\nu_\rho \rightarrow \nu_\sigma}(t). \quad (32)$$

with  $\Delta_{CP}(t) = \Delta_T(t)$  as a consequence of CPT invariance.

The QFT analogue of Eq.(31) is

$$\Delta_{CP}^{\rho\sigma}(t) \equiv \mathcal{Q}_\sigma^\rho(t) + \mathcal{Q}_\sigma^{\bar{\rho}}(t) \quad , \quad \rho, \sigma = e, \mu, \tau. \quad (33)$$

with  $\sum_\sigma \Delta_{CP}^{e\sigma}(t) = 0$  since  $\sum_\sigma Q_\sigma(t) = Q$  and  $\langle \nu_e | Q | \nu_e \rangle = 1$ ,  $\langle \bar{\nu}_e | Q | \bar{\nu}_e \rangle = -1$ . For the case of an initial electron neutrino state, we obtain

$$\begin{aligned} \Delta_{CP}^{e\mu}(t) = & \frac{1}{2} \cos \theta_{13} \sin \delta \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \left[ U_{12}^2 \sin(2\Delta_{12}^{\mathbf{k}} t) \right. \\ & - V_{12}^2 \sin(2\Omega_{12}^{\mathbf{k}} t) - U_{13}^2 \sin(2\Delta_{13}^{\mathbf{k}} t) + V_{13}^2 \sin(2\Omega_{13}^{\mathbf{k}} t) \\ & \left. + (U_{12}^2 - V_{13}^2) \sin(2\Delta_{23}^{\mathbf{k}} t) + (V_{12}^2 - V_{13}^2) \sin(2\Omega_{23}^{\mathbf{k}} t) \right] \end{aligned} \quad (34)$$

and  $\Delta_{CP}^{e\tau}(t) = -\Delta_{CP}^{e\mu}(t)$ .

For the study of T violation, defining  $\Delta_T(t)$  as  $\Delta_T^{\rho\sigma}(t) \equiv \mathcal{Q}_\sigma^\rho(t) - \mathcal{Q}_\rho^\sigma(t)$  does not seem to work. We have indeed:  $\Delta_T^{\rho\sigma}(t) - \Delta_{CP}^{\rho\sigma}(t) \neq 0$  violating CPT invariance. The correct definition is then:

$$\Delta_T^{\rho\sigma}(t) \equiv \mathcal{Q}_\sigma^\rho(t) - \mathcal{Q}_\sigma^\rho(-t) \quad , \quad \rho, \sigma = e, \mu, \tau. \quad (35)$$

We obtain  $\Delta_T^{e\sigma} \neq 0$  for  $\sigma \neq e$  and  $\Delta_T^{e\sigma} = \Delta_{CP}^{e\sigma}$  in agreement with  $\mathcal{Q}_\sigma^e(-t) = -\mathcal{Q}_\sigma^e(t)$ . Further work is in progress on this topic.

## 6 Conclusions

We have discussed the mixing of (Dirac) fermionic fields in Quantum Field Theory for the case of three flavors. We have constructed the flavor Hilbert space and studied the currents and charges for mixed fields (neutrinos).

We have then derived the exact QFT oscillation formulas, a generalization of the usual QM Pontecorvo formulas. CP and T violation induced by neutrino oscillations have also been discussed.

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